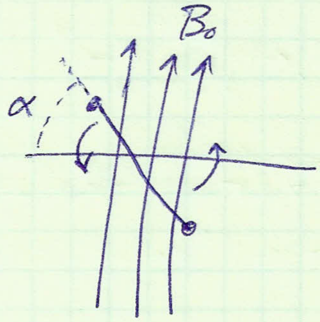
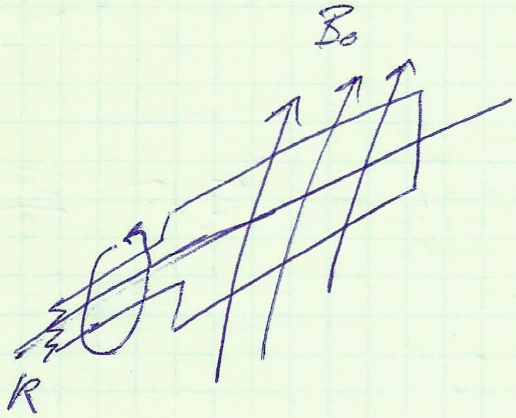
Electromagnetic Generator





$$V_{emf} = -N\frac{d\Phi}{dt}$$

Area of loop = A

$$\Phi = \int \vec{B}\cdot d\vec{s} = B_0 A \cos\alpha$$

$\alpha = \omega t \leftarrow$ rotating with frequency ω

$$\underline{V_{emf} = NAB_0\omega \sin\omega t}$$

General Case:

Moving conductor in time-varying magnetic field

$$V_{emf} = V_{emf}^{tr} + V_{emf}^{m} = \oint_c \vec{E}\cdot d\vec{l}$$

$$= -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s} + \oint_c (\vec{u}\times\vec{B})\cdot d\vec{l}$$

or just $V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_s \vec{B}\cdot d\vec{s}$ (total)

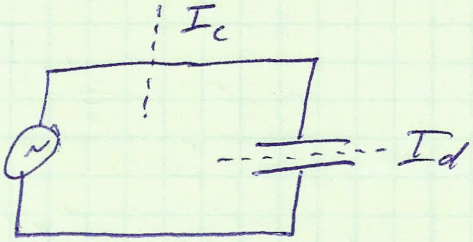
Displacement Current

Ampere's law: $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \leftrightarrow \oint_c \vec{H} \cdot d\vec{l} = I_c + \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$

$\uparrow$ conduction current

displacement current $I_d = \int_s \vec{J}_d \cdot ds = \int_s \frac{\partial \vec{D}}{\partial t} \cdot ds$

$J_d = \frac{\partial D}{\partial t}$ = displacement current density



$I = I_c + I_d$

- Boundary Conditions for Electromagnetics

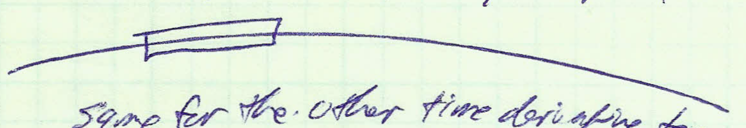
$\hat{n}_2 \times (\vec{E}_1 - \vec{E}_2) = 0$

$\hat{n}_2 \cdot (\vec{D}_1 - \vec{D}_2) = \rho_s$

$\hat{n}_2 \times (\vec{H}_1 - \vec{H}_2) = J_s$

$\hat{n}_2 \cdot (\vec{B}_1 - \vec{B}_2) = 0$

$\nabla \times E = \frac{-\partial B}{\partial t} \leftarrow$ this term goes to 0 for very thin path



same for the other time derivative terms

The boundary conditions do not change from electrostatics, magnetostatics

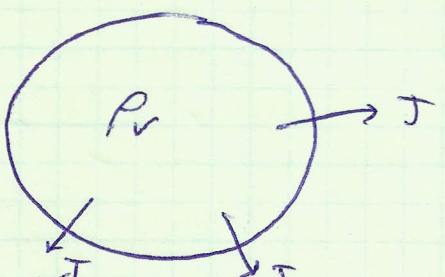
- Charge-current continuity relation

$I = \frac{-dQ}{dt} = \frac{-d}{dt} \int_v \rho_v \, dv$

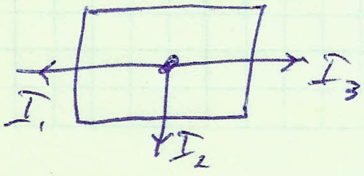
$\oint_s \vec{J} \cdot d\vec{s} = \frac{-d}{dt} \int_v \rho_v \, dv$

$\int_v \nabla \cdot \vec{J} \, dv = \frac{-d}{dt} \int_v \rho_v \, dv$ divergence theorem

$\int_v \nabla \cdot \vec{J} \, dv = -\int_v \frac{d\rho_v}{dt} \, dv$ stationary volume



$\boxed{\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t}}$ when $\frac{\partial \rho_v}{\partial t} = 0 \rightarrow \oint_s J \cdot ds = 0$: Kirchoff's current law



$\sum_i I_i = 0$

- Free-Charge dissipation in a conductor

$$\nabla\cdot\vec{J} = -\frac{\partial\rho_v}{\partial t}$$

$$\vec{J} = \sigma\vec{E} \Rightarrow \sigma\nabla\cdot\vec{E} = -\frac{\partial\rho_v}{\partial t}$$

$$\nabla\cdot\vec{E} = \frac{\rho_v}{\epsilon} \Rightarrow \frac{\partial\rho_v}{\partial t} + \frac{\sigma}{\epsilon}\rho_v = 0$$

examples:

copper: $\tau_r = 10^{-19}$ seconds

mica: $\tau_r = 15$ hours

at time $t=0$, $\rho_v(t) = \rho_{v0}$

$$\rho_v(t) = \rho_{v0}e^{-\left(\frac{\sigma}{\epsilon}\right)t} = \rho_{v0}e^{-t/\tau_r}$$

$\tau_r = \frac{\epsilon}{\sigma}$ relaxation time constant

the time for free charges to dissipate (assuming connection to ground)

- Electromagnetic Potentials

Electrostatics : $\nabla\times\vec{E} = 0 \rightarrow \vec{E} = -\nabla V$

Dynamics : $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$, $\vec{B} = \nabla\times\vec{A} \rightarrow \nabla\times\vec{E} = -\frac{\partial}{\partial t}(\nabla\times\vec{A})$

$$\nabla\times\left(\vec{E} + \frac{\partial\vec{A}}{\partial t}\right) = 0$$

$$\vec{E} = -\nabla V - \frac{\partial\vec{A}}{\partial t} \quad \text{(remember } \nabla\times\nabla V = 0\text{)}$$

$$\vec{B} = \nabla\times\vec{A}$$

$\left\{\begin{array}{l}\text{define } E' = E + \frac{\partial A}{\partial t} \\ \nabla\times E' = 0 \\ E' = -\nabla V\end{array}\right.$

- Retarded potentials

Electrostatics : $V(\vec{R}) = \frac{1}{4\pi\epsilon}\int_{v'} \frac{\rho_v(\vec{R}_i)}{R'}dv'$

Dynamics : $V(\vec{R},t) = \frac{1}{4\pi\epsilon}\int_{v'} \frac{\rho_v(\vec{R}_i, t - R'/u_p)}{R'}dv'$

$$A(\vec{R},t) = \frac{\mu}{4\pi}\int_{v'} \frac{\vec{J}(\vec{R}_i, t - R'/u_p)}{R'}dv'$$

u_p : propagation velocity in medium

The effect of charges or currents can only be felt at a delayed time

The speed of the delay is the speed of light

Time-harmonic potentials

$$\rho_v(\vec{R_i},t) = \rho_v(\vec{R_i})\cos(\omega t + \phi) = \mathrm{Re}\left[\tilde{\rho}_v(\vec{R_i})e^{j\omega t}\right]$$

retarded charge density

$$\rho_v(\vec{R_i}, t - R'/u_p) = \mathrm{Re}\left[\tilde{\rho}_v(\vec{R_i})e^{-j\omega R'/u_p}e^{j\omega t}\right] = \mathrm{Re}\left[\tilde{\rho}_v(\vec{R_i})e^{-jkR'}e^{j\omega t}\right]$$

$k = \frac{\omega}{u_p}$ wavenumber (different notation for β)

$$V(\vec{R},t) = \mathrm{Re}\left[\tilde{V}(\vec{R})e^{j\omega t}\right] = \mathrm{Re}\left[\frac{1}{4\pi\epsilon}\int_{V'}\frac{\tilde{\rho}_v(\vec{R_i})e^{-jkR'}}{R'}e^{j\omega t}\,dV'\right]$$

$$\tilde{V}(\vec{R}) = \frac{1}{4\pi\epsilon}\int_{V}\frac{\tilde{\rho}_v(\vec{R_i})e^{-jkR'}}{R'}\,dV'$$ ← phasor domain expression

$$\tilde{\vec{A}}(\vec{R}) = \frac{\mu}{4\pi}\int_{V'}\frac{\tilde{\vec{J}}(\vec{R_i})e^{-jkR'}}{R'}\,dV'$$ similar expression for A

$$\nabla\times\tilde{\vec{H}} = \frac{\partial\vec{D}}{\partial t} + \tilde{\vec{J}} \Longrightarrow \nabla\times\tilde{\vec{H}} = j\omega\epsilon\tilde{\vec{E}}$$ for nonconducting, $J=0$

$$\nabla\times\tilde{\vec{E}} = \frac{\partial\vec{B}}{\partial t} \Longrightarrow \nabla\times\tilde{\vec{E}} = -j\omega\mu\tilde{\vec{H}}$$